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The BenImpact Demand System

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A demand system for the BenImpact model

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1 Introduction

BenImpact is a model of the agricultural sector of the West-African country Benin. In Benin, there is a high degree of subsistence farming. Most households are engaged in agriculture in some way. In the IFPRI smallholder survey of Benin, 93 % of the households had income from crop sales (IFPRI 2001), and are hence both supplying and demanding agricultural goods. Agriculture is the single most important source of income for most households (ibid.). Therefore, the model is formulated as a household model as in Singh, Squire and Strauss (1986), with endogenous budget constraint. Decisions are taken in two steps. In the first step, production is planned to fit given demand quantities and labour supply. This is done in a constrained quadratic mathematical programming model. The dual values from the market balances of that step are the regional prices, and using them, household income is computed. In the second step, prices and income are held fixed and demanded quantities are calculated. The model iterates between those steps until equilibrium is reached. It is necessary to split up the procedure into two steps because of the assumption that each household is small, not influencing market equilibrium, and that the production decision is taken independent of the consumption decisions (and vice versa).

In this paper, the demand system component is outlined and implemented. It is based on the demand module of the CAPRI modelling system (Britz 2004), but is modified to consider temporal separability.

2 Overall demand system layout

The exogenous values of the demand system in BenImpact are income and prices, and the endogenous ones are demanded quantities of goods. The consumers are thought of as solving the following utility maximisation problem:

$$\max_x U(x) \;\; s.t. \;\; px = m$$

The vector of consumed goods is x, the price vector for consumption goods is p and m is income from production and other sources.

We want to simulate demand for n goods in T periods in a comparative static model. There is one "observation" of demand $x=\{x^1,\dots,x^T\}$ where x^t is the column vector of demands for goods $i=\{1,...,n\}$ in period t, and the corresponding price matrix $p=\{p^1,\dots,p^T\}$ with row vectors p^t similarly defined. Furthermore, there is a-priori information on the matrix of Marshallian demand elasticities e, but only for the year as a whole, i.e. there is no information available for cross price elasticities of the type "what would happen to consumption of product i in period 1 if the price of product j in period 2 would change?".

The system shall be able to deliver the demand matrix at a given price matrix and given annual income. Furthermore, it must be flexible enough to be able to reproduce exactly the observed demand at observed prices and income, and should satisfy the following properties that ensure that there exists some utility function from which the conditional demand functions can be derived:

1. Monotonicity: Expenditure monotonously increasing in prices $\Leftrightarrow$ Indirect utility function monotonously decreasing in prices.

2. No money illusion $\Leftrightarrow$ indirect utility function homogeneous of degree zero in prices and income.

3. The indirect utility function is convex in prices $\Rightarrow$ we require the Hessian to be symmetric and positive semi definite.

One approach would be to assume some explicit utility function that is maximised subject to a budget constraint spanning all periods. As long as the function is convex and monotonously increasing in consumption, all conditions above are satisfied. However, not all such functions are flexible enough to fit the given observation and still have some degrees of freedom left to match the desired elasticity matrix.

Another approach is to assume a demand function that is flexible enough to match the observed point and enforce conditions 1-3 in a calibration (estimation) step. Ryan and Wales (1996) propose to use demand functions based on indirect utility functions of the form

$$v(p,m) = -\frac{g(p)}{m - f(p)} - h(p) \tag{1}$$

where m is income and g, f and h are functions of prices and unknown parameters. In the following, those functions (but no other) will be referred to without list of arguments, and their derivatives with respect to the price of good i will be denoted by a subscript i (and two subscript for second order derivatives). Ryan and Wales select

$$f = \sum_k p_k d_k$$

$$h = \sum_k a_k \log p_k \quad with \quad \sum_k a_k = 0,$$

and propose, for a Generalised Leontief expenditure system, to use

$$g = \sum_k p_k b_k + \sum_k \sum_j B_{kj} \sqrt{p_k p_j}$$

with B_{ij}=B_{ji}. We want to keep the number of parameters to determine at a minimum. Thus, we select a_k=0 for all k, rendering h=0 for any price vector. We also simplify the expression for g by adding b_i to the diagonal elements B_{ii}. Both f and g are homogeneous of degree one, so the indirect utility function (1) is homogeneous of degree zero in prices and income, satisfying point 2 above by construction. Furthermore, if we are prepared to accept that no Hicksian complementarities are allowed between goods, we may achieve requirements 1 and 3 by restricting all off-diagonal elements of B to be non-negative (Diewert and Wales 1987).

For our specific case, the products are not only distinguished by kind, but also by time of consumption. It seems reasonable to assume that consumption in one period does not influence marginal utility of consumption in another period, but that intertemporal price effects arise solely through the working of the budget constraint. As the model is comparative static, there is no interest rate, so it is also does not seem too abstruse an assumption that the marginal utility of income will be the same in each period. This would imply that the consumer (household) is acting as solving the problem

$$\max_x \sum_t U^t\left(x^t\right) \ s.t. \ \sum_t \left(p^t x^t\right) = m,$$

where there is a utility function U_t in each period, each potentially different, and that the sum of utilities is maximised. This implies functional separability of consumption in different time periods.

Proposition: If B_{ij}=0 whenever i is consumed in another time period than j, then substitution effects between periods are zero, whereas income effects still are possible.

Proof: By explicitly deriving the Slutsky equation, the result follows.

The Slutsky equation with net demands is

$$\frac{\partial x_j(p,m)}{\partial p_i} = \frac{\partial h_j(p, v(p,m))}{\partial p_i} - \frac{\partial x_j(p,m)}{\partial m} x_i \qquad (2)$$

with $h(p,u)$ being the compensated demand equations. Using (1), we can use Roy's identity to derive Marshallian demands $x(p,m)$, solve (1) for income to get the expenditure function $e(p,u)$, and use the identity $h(p,u) \equiv x(p,e(p,u))$ to derive compensated demand functions. Taking the partial derivatives, some manipulations give the two terms of the Slutsky equation

$$\frac{\partial h_j(p, v(p,m))}{\partial p_i} = \frac{-g_{ji}}{u^*} + f_{ji} \tag{3}$$

$$\frac{\partial x_j(p,m)}{\partial m} x_i = \frac{g_j}{g}\left(\frac{g_i(m-f)}{g} + f_i\right) \tag{4}$$

Using the proposed definitions of g and f, we have that

$$\begin{aligned} f_i &= d_i \\ f_{ij} &= 0 \\ g_i &= \sum_j B_{ij} p_j^{\frac{1}{2}} p_i^{-\frac{1}{2}} \end{aligned} \tag{5}$$

(6)

Thus, f_{ij} is always zero, and equation (6) shows that g_{ij} is zero if B_{ij} is zero. So the substitution effect (3) is also zero, whereas there is nothing that requires the income effect (4) to disappear, because g_j contains the sum over an entire row of B. □

If the elements of B are ordered according to time of consumption, the B matrix constructed in this way will be block-wise diagonal. In the following, we write it as a cube with a time index on one of the axes.

3 Calibration of the GL expenditure system

The demand functions, deducted from the indirect utility function using Roy's identity, are

$$x_i(p,m) = \frac{g_i(m-f)}{g} + f_i$$

From these, we compute the first order partial derivatives w.r.t. prices,

$$\frac{\partial x_i(p,m)}{\partial p_j} = \frac{g_{ij}(m-f)}{g} - \frac{g_j f_i}{g} - \frac{g_i g_j (m-f)}{g^2}$$

the conditional demand elasticities at observed (calibrated) prices and quantities *pobs*, *xobs*

$$\varepsilon_{ij} = \frac{\partial x_i(p,m)}{\partial p_j} \frac{p_j^{obs}}{x_i^{obs}}$$

and income elasticities at observed income *mobs*

$$\varepsilon_{im} = \frac{\partial x_i(p,m)}{\partial m}\frac{m^{obs}}{x_i^{obs}}$$

Using these equations, we can put together the following estimation problem for each region, where time indices have been added to distinguish between products consumed in different periods and the sets i and j have been changed accordingly.

$$\min_{B,d} \sum_t \left(\sum_i \left(\sum_j \left(\varepsilon_{itjt} - e_{ij}\right)^2 + \left(\varepsilon_{itm} - e_{itm}\right)^2 \right)\right) \tag{7}$$

subject to symmetry of B,

$$B_{tij} = B_{tji}$$

non-negativity of non-diagonal elements of B,

$$B_{tij} \geq 0 \quad \forall i \neq j$$

and that minimum demand of any product is zero.

$$d_{it} \geq 0 \tag{8}$$

Note that equation (7) only penalizes deviations of elasticities within each time period from the a-priori information, whereas intertemporal cross price elasticities do not enter into the objective. However, such effects are limited because the substitution terms B_{ij} are zero if i is consumed in another time period than j. As the price of i tends to infinity, demand for i tends to d_i, so equation (8) is required to avoid that negative consumption is admitted.

4 Data

The calibration process requires data on quantities and prices, and a-priori information on demand elasticities.

Data on consumed quantities and prices come from a calibration process previously executed. Elasticities of demand were compiled from different sources in a hierarchical way, taking firstly data from Beck (1995), then, for positions still missing, from Senahoun (2000), then from the FAO World Food Model (WFM). For elasticities still missing after this process, it was assumed that own price elasticity is -0.7, cross price elasticity within the same time period is 0.05, and income elasticity is 0.5.

The data from Beck (1995) and Senahoun (2000) is regionalised to Agro-ecological zones, of which there are nine in Benin, whereas the WFM data does not distinguish any sub regions of the country. In the table below the resulting table of a-priori elasticities for Benin are shown. The numbers have been computed from the regional numbers using population shares as weights. Own price elasticities are printed in bold numbers to facilitate reading.

Table 5.1. A-priori elasticities of demand aggregated to Benin country level from market regions using population shares as weights.

	CASS	MAIZ	NIEB	SORG	YAMS	PEAN	RICE	REST	INCE
CASS	**-0.734**	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.500
MAIZ	0.050	**-0.634**	0.050	0.020	0.050	0.050	0.040	0.050	0.300
NIEB	0.050	0.050	**-0.650**	0.050	0.050	0.050	0.050	0.050	0.500
SORG	0.050	0.080	0.050	**-0.634**	0.050	0.050	0.030	0.050	0.400
YAMS	0.050	0.050	0.050	0.050	**-0.734**	0.050	0.050	0.050	0.500
PEAN	0.050	0.050	0.050	0.050	0.050	**-0.600**	0.050	0.050	0.500
RICE	0.050	0.080	0.050	0.020	0.050	0.050	**-0.750**	0.050	1.200
REST	0.050	0.050	0.050	0.050	0.050	0.050	0.050	**-0.700**	0.500

Source: Compiled from Beck (1995), Senahoun (2000), FAO World Food Model and own assumptions.
CASS=Cassava, NIEB=Niebe, SORG=Sorghum, PEAN=Peanuts, REST=All other products, INCE=Income.

5 Results

The resulting calibrated elasticities are shown in table 6.1 below. That data has been aggregated from market regions to country level just for the purpose of exposition, using population shares as aggregation weights. To save space, only reactions to changing the prices in period P1 and income (INCE) are displayed. The numbers in the upper part of the table correspond to the a-priori information in table 5.1.

Table 6.1. Calibrated elasticities of demand. Reaction to price changes of the goods in the columns in period P1 and to annual income change (INCE). Data aggregated from market regions to country level using population shares as weights.

	CASS	MAIZ	NIEB	SORG	YAMS	PEAN	RICE	REST	INCE
P1.CASS	**-0.639**	0.073	0.025	0.056	0.059	0.015	0.029	0.186	0.718
P1.MAIZ	0.123	**-0.591**	0.037	0.050	0.044	0.027	0.051	0.125	0.486
P1.NIEB	0.088	0.065	**-0.633**	0.060	0.070	0.052	0.066	0.084	0.542
P1.SORG	0.092	0.073	0.040	**-0.601**	0.081	0.033	0.027	0.110	0.534
P1.YAMS	0.090	0.061	0.050	0.060	**-0.672**	0.043	0.055	0.135	0.648
P1.PEAN	0.074	0.064	0.064	0.054	0.060	**-0.594**	0.065	0.067	0.531
P1.RICE	0.082	0.081	0.049	0.027	0.051	0.037	**-0.744**	0.073	1.247
P1.REST	0.041	0.009	0.003	0.011	0.024	0.000	0.007	**-0.480**	1.316
P2.CASS	-0.029	-0.019	-0.005	-0.015	-0.018	-0.004	-0.007	-0.141	0.727
P2.MAIZ	-0.019	-0.013	-0.003	-0.010	-0.011	-0.003	-0.004	-0.096	0.482
P2.NIEB	-0.020	-0.012	-0.004	-0.011	-0.014	-0.003	-0.005	-0.110	0.539
P2.SORG	-0.020	-0.013	-0.004	-0.012	-0.014	-0.003	-0.005	-0.103	0.527
P2.YAMS	-0.024	-0.013	-0.005	-0.014	-0.020	-0.004	-0.006	-0.128	0.643
P2.PEAN	-0.020	-0.012	-0.004	-0.011	-0.014	-0.003	-0.005	-0.109	0.532
P2.RICE	-0.046	-0.029	-0.009	-0.025	-0.032	-0.007	-0.011	-0.258	1.251
P2.REST	-0.033	-0.023	-0.006	-0.016	-0.018	-0.004	-0.007	-0.170	0.827
P3.CASS	-0.030	-0.020	-0.005	-0.015	-0.019	-0.004	-0.007	-0.144	0.748
P3.MAIZ	-0.019	-0.013	-0.003	-0.010	-0.011	-0.003	-0.004	-0.095	0.479
P3.NIEB	-0.020	-0.012	-0.004	-0.011	-0.014	-0.003	-0.005	-0.110	0.538
P3.SORG	-0.020	-0.013	-0.004	-0.012	-0.014	-0.003	-0.005	-0.102	0.524
P3.YAMS	-0.024	-0.013	-0.005	-0.014	-0.021	-0.004	-0.006	-0.130	0.652
P3.PEAN	-0.020	-0.012	-0.004	-0.011	-0.014	-0.003	-0.005	-0.109	0.530
P3.RICE	-0.046	-0.029	-0.009	-0.025	-0.032	-0.007	-0.011	-0.258	1.250
P3.REST	-0.030	-0.020	-0.006	-0.016	-0.018	-0.004	-0.007	-0.157	0.786
P4.CASS	-0.031	-0.020	-0.006	-0.016	-0.019	-0.004	-0.007	-0.146	0.765
P4.MAIZ	-0.019	-0.013	-0.003	-0.010	-0.011	-0.003	-0.004	-0.096	0.484
P4.NIEB	-0.020	-0.012	-0.004	-0.011	-0.014	-0.003	-0.005	-0.110	0.540
P4.SORG	-0.020	-0.012	-0.004	-0.012	-0.014	-0.003	-0.005	-0.102	0.524
P4.YAMS	-0.023	-0.013	-0.005	-0.014	-0.020	-0.003	-0.006	-0.127	0.635
P4.PEAN	-0.020	-0.012	-0.004	-0.011	-0.014	-0.003	-0.005	-0.109	0.530
P4.RICE	-0.046	-0.029	-0.009	-0.025	-0.032	-0.007	-0.011	-0.257	1.248
P4.REST	-0.034	-0.018	-0.009	-0.021	-0.031	-0.006	-0.009	-0.178	0.999

Source: Calibration of the BenImpact model, as of December 2004. Estimation may improve as better data on demand and regional prices become available. Note that most of the cross price effects between periods are omitted.

It can be seen that all the own price and income elasticities have been adjusted in the positive direction. This is happening because of the requirement to satisfy the microeconomic conditions referred to in the beginning of this paper. The homogeneity condition requires the row sum of price and income elasticities for each product equal zero, i.e. increasing all prices and income by the same factor does not provoke any change in consumption. Because of the temporal functional separability, the cross-period price effects are all negative – only the income effect of the Slutsky equation is active, as previously shown. This can be seen in the lower part of table 6.1 (the columns for price changes of goods in other periods has been left out in order to save space), where all the price elasticities are negative. The row sums of table 5.1 are already rather close to zero, so in order to make place for all the negative cross-time terms, the estimator tends to shift elasticities upwards.

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