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The Economic Model BenImpact

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1 Model overview

BenImpact is an agricultural household model of Benin, developed at the institute for agricultural policy of Bonn University, Germany. The model consists of separate supply and demand models that are solved iteratively. Supply is modelled by regional aggregate farm models, implemented with quadratic programming. Demand is modelled by a generalized Leontieff expenditure system (Ryan and Wales 1996). The income generated in the supply model enters the demand model as exogenous values.

2 Supply model

Supply is modelled on the regional level of departments, meaning that there are 12 supply regions in Benin. Each region is represented by an aggregated quadratic farm model. In the current version of the model, the supply of eight important crops is modelled: cassava, cotton, maize, pulses, sorghum and millet, yams, peanuts and rice. For all crops except cotton, different varieties (local/commercial varieties, early/late season) are modelled. The function to maximize is the value of net trade with the world market plus cotton sales to factories minus costs for storage, transportation and inputs (eq. 1). The model has a land constraint (eq. 4), but it is not binding in the reference period. Instead, resource scarcity is modelled by quadratic cost terms in the objective function (the second term from the end of equation 1).

The supply of neighbour countries (Nigeria, Niger, Togo and Burkina Faso) is modelled by maximising a function that is quadratic in output without any technical constraints. The function is included as the last term in the objective function (eq. 1). The quadratic cost implies linear supply curves for the neighbour countries.

All regional models are connected via market balances, requiring that regional demand, which is exogenous to the supply model, is satisfied in all regions for all products (eq. 3). The model is spatial, with bi-lateral net trade flows between all regions. The model has four time periods in the calendar year, enabling the modelling of two growing seasons and analyses of income distribution within a year. Storage at fixed costs between time periods is allowed.

Some regions have cotton processing plants. In those regions, raw cotton for processing is demanded at fixed prices up to the factory capacity limit, modelled for the entire year (eq. 5). The exogenous variables in the supply model are de-

mand quantities H (for human consumption) in each region and period for each product. Endogenous are regional production, trade, storage and prices. Prices are obtained as dual values λ of the market balance equations.

$$\begin{aligned} \max \sum_{jot} (T_{j'row'ot} - T_{row'jot}) P_{row'ot} + \sum_{jot} F_{jot} P_{jot}^r \\ - \sum_{jot} S_{jot} SC_{jo} - \sum_{jkot} T_{jkot} TC_{jkot} - \sum_{dit} N_{dit} P_{dit} \end{aligned} \quad (1)$$

$$- \sum_{du} X_{du} \left(A_{du} + \frac{1}{2} \sum_v B_{duv} X_{dv} \right) - \sum_{not} N_{not} \left(a_{not} + \frac{1}{2} b_{not} N_{not} \right)$$

$$\text{subject to } N_{dgt} = \sum_{dv} X_{dv} IO_{dvgt} \quad (2)$$

$$N_{jot} + \sum_k T_{kjot} - \sum_k T_{jkot} + S_{jot-1} - S_{jot} - F_{jot} \geq H_{jot} \quad [\lambda_{jot}] \quad (3)$$

$$\sum_{ju} X_{ju} l_{jut} \leq L_{jut} \quad (4)$$

$$\sum_t F_{jot} \leq K_{jo} \quad [\pi_{jo}] \quad (5)$$

where j,k = all regions except rest of world 'row'

$d \subset j$ = domestic regions

$n \subset j$ = neighbour countries

g = goods in the model

$o \subset g$ = outputs

$i \subset g$ = inputs

u,v,w = production activities

t = time periods 1 to 4

T = transport flows

TC = transport cost

P = prices of goods

P^r = regulated price at Factory

F = processing consumption

S = storage

SC	= storage cost
N	= net sales of farm
X	= activity levels
l	= land use of activity in periods
L	= land availability
K	= maximum factory capacity (only for cotton)
A	= linear parameter of cost function for domestic regions
B	= quadratic parameter of cost function for domestic regions
a	= linear parameter of cost function for neighbour countries
b	= quadratic parameter of cost function for neighbour countries

The model is calibrated to the year 2001. In addition to balancing physical quantities, calibration requires that the first order conditions hold for that year. This implies that (i) regional price differences are equal to (where trade takes place) or less than trading costs, (ii) price differences between time periods are equal to (where storage takes place) or less than storage costs, (iii) the first derivatives of the quadratic terms for domestic regions of (eq. 1) equal gross margin minus dual values of fixed resources (land and factory capacity for cotton), and (iv) the first derivatives of the quadratic terms for neighbour countries equal prices. This corresponds to, in turn, the derivatives of the Lagrangean with respect to trade flows, storage, activity levels in domestic regions and net production in neighbour countries.

The first two derivatives (i) and (ii) do not contain any terms of the quadratic functions, and can thus be estimated together but separate from (iii) and (iv). This was done by a least squares estimation of the Kuhn-Tucker conditions. The complementarity conditions make the problem non-convex and require special solution methods for the estimation. For BenImpact, this was done by stepwise smooth approximations as in Facchinei, Jiang and Qi (1999) and Ferris, Dirkse, Meeraus (2002).

When prices, storage and transport costs have been estimated, the quadratic functions can be calibrated. For (iii), the parameters A and B were computed using exogenous elasticities of supply. This was done region-wise with the estimation given by equations 6 to 12, minimising the squared deviation of estimated elasticities of supply from given own-price elasticities and squared deviation of total land use elasticity (to all agricultural prices) from a given amount (eq. 6), subject to the conditions that the matrix B for each region is positive semi definite (using

a Cholesky factorization, eq. 7, see e.g. Greene 2003), and that marginal revenue equals marginal cost (eq. 8).

$$\min(\varepsilon^a - e^a)^2 + \sum_u (\varepsilon_{uu}^s - e_{uu}^s)^2 \quad (6)$$

$$B_{uv} = \sum_w U_{vw} U_{uw} \quad (7)$$

$$GM_u = A_u + \sum_v B_{uv} X_v \quad (8)$$

$$\varepsilon_{uv}^s = B_{uv}^{-1} \frac{R_v}{X_u} \quad (9)$$

$$\varepsilon^a = \frac{\sum_{uv} (\varepsilon_{uv}^s X_u)}{\sum_u X_u} \quad (10)$$

$$\sum_w B_{uw}^{-1} B_{wv} = I_{uv} \quad (11)$$

$$B_{uv} = \beta \quad \forall u \neq v \quad (12)$$

where ε_{uv}^s = elasticity of supply of activity u to revenues of v

ε^a = elasticity of supply of total crop area to all agricultural prices

e = values for elasticities from literature (except e^a , assumed 0.6)

U = an upper triangular matrix

GM = gross margin minus dual values of resources

R = revenues per hectare

I = identity matrix of appropriate dimensions

Equation (12) ensures that marginal costs of all activities increase with the same amount if total area use is increased, i.e. the quadratic term of (eq. 1) for a single domestic region can be reformulated as

$$\sum_u X_u \left(A_u + \frac{1}{2} \sum_v B_{uv} X_v \right) = \sum_u \left(X_u A_u + \frac{1}{2} \delta_u (X_u)^2 \right) + \frac{1}{2} \beta \left(\sum_u X_u \right)^2$$

with $B_{uu} = \delta_u + \beta$ and $B_{uv} = \beta$ for all $u \neq v$.

For the neighbour countries the calibration is easy, as there is no land constraint and no cross effects. The terms of the quadratic function are computed to

recover a given point elasticity of supply at the 2001 prices and quantities. The computation is trivial and not documented here.

3 Demand model

The exogenous values of the demand system in BenImpact are income and prices, and the endogenous ones are demanded quantities of goods. The consumers are thought of as solving the following utility maximisation problem:

$$\max_x \sum_t U^t(H^t) \text{ s.t. } \sum_t (P^t H^t) = Y$$

The vector of consumed goods is Q , the price vector for consumption goods is P and Y is income from production and other sources. Note that there is a utility function U_t in each period, each potentially different, and that the sum of utilities is maximised. This implies functional separability of consumption in different time periods. There is only a single budget restriction, implying zero interest rate within a year.

In BenImpact, this is implemented, following Ryan and Wales (1996) with a Generalized Leontieff (GL) expenditure system, where the demand functions are of the form

$$H_o(P, Y) = \frac{g_o(Y - f)}{g} + f_o$$

where g and f are functions that are homogeneous of degree one in prices and g_i the derivative of g with respect to good o . For a more thorough motivation and technical details about time separability of consumption and calibration issues, see Jansson 2005.

Exogenous variables to the demand system are prices P and income Y , endogenous are the regional demand quantities H .

4 Iterative solution

The supply model is a profit maximisation problem whereas the demand model is a system of behavioural equations to solve. Therefore, the supply and demand cannot be solved simultaneously. That would let the producers behave like monopolists, which is implausible for the small-structured farming sector of Benin. Instead, the model is solved iteratively, so that demand quantities are considered fixed in the supply model.

To make the model converge, two mechanisms are implemented. The first mechanism is that instead of taking the prices from the supply model directly into the demand model, an average price of several preceding iterations is formed. The second mechanism is a set of linear supply functions that are implemented inside the market model, connecting prices to quantities. In each step, those functions

are calibrated to the last solution of the true supply model, and are then solved together with the demand system, with endogenous prices. In that way, the demand model approximates the outcome of the next supply step.

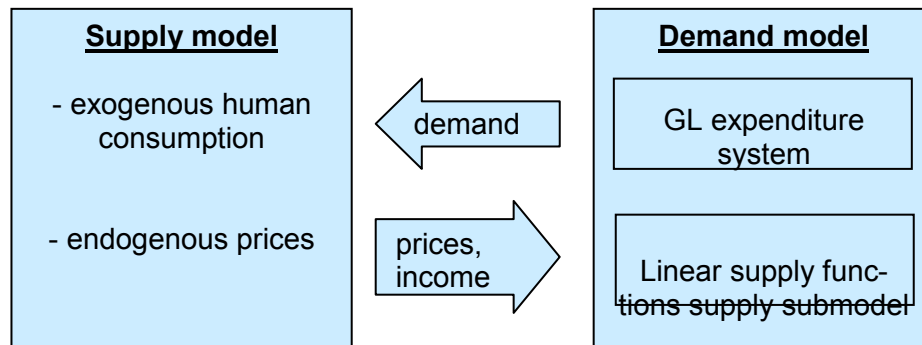


Figure 1: Overview over iterations in BenImpact.

Income is computed as sum of revenues minus variable costs, plus 50% of trade costs, assumed to accrue in equal parts to exporting and importing region, plus 50% of storage cost, plus marketing margin in the region where final demand takes place, plus income from other sectors, assumed to be constant.

5 Literature

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