DFG Forschergruppe 1501

Subproject B2

10/2012

An Earth Observation- and Integrated Assessment (EOIA) Approach to the Governance of Lake Naivasha, Kenya

Arnim Kuhn[1] and Pieter R. van Oel[2], Frank M. Meins[2]

The Lake Naivasha Hydro-Economic Basin Model (LANA-HEBAMO)

- Technical Documentation -

Address:

[1]Institute for Food and Resource Economics

University of Bonn

Nussallee 21

D - 53115 Bonn

Tel.: 0049-228-73 2843

Internet: http://www.fg1501.uni-koeln.de/

http://www.ilr1.uni-bonn.de/agpo/rsrch/RCR/RCR_e.htm

[2]ITC - University of Twente

P.O. Box 217

7500 AE Enschede

The Netherlands

www.itc.nl

1 Background: The Lake Naivasha Basin

Lake Naivasha is the second largest fresh water lake in Kenya and a Ramsar site located in the Rift Valley (*0^0 45′ S, 36^0 20′ E*) with a basin approximating 3400 Km^2 (Figure 1). The Lake basin can be viewed as a social-ecological system (SES) with strong interdependent feedback mechanisms. The basin ecosystem is composed of an endorheic fresh water system that feeds a lake system that consists of a main lake (Lake Naivasha), a semi-separated sodic extension (Oloiden Lake) and a separate sodic crater Lake (Sonachi). The inflow into the main lake comes from the Malewa, Gilgil and Karati rivers.. The main Lake is a freshwater wetland with fringing shoreline vegetation dominated by floating and submerged swamp species, e.g. *Cyperus papyrus* (Harper & Mavuti, 2004). The river delta vegetation plays an important role in regulating incoming materials such as dissolved and/or suspended nutrients and sediments.

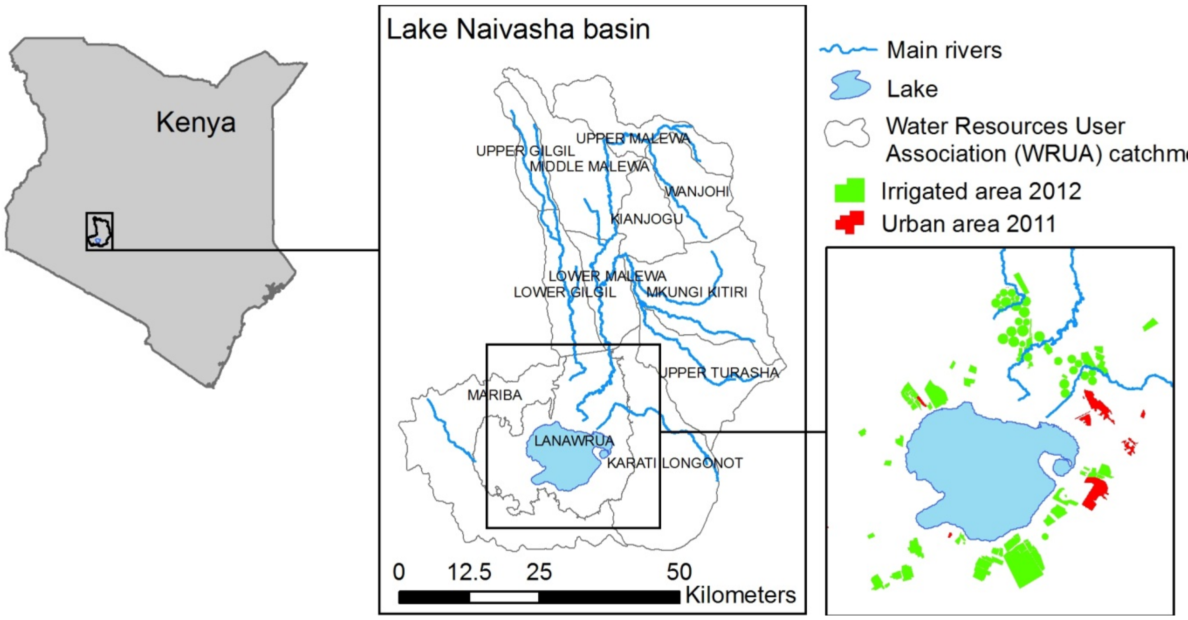

Figure 1: Lake Naivasha basin showing the 12 Water Resource Users Associations and urbanized and irrigated area directly around the lake.

The RAMSAR Convention (2011) describes the Lake Naivasha ecosystem as very rich in biodiversity since it provides habitat for a wide range of terrestrial flora and fauna and aquatic organisms which all play an important role in sustaining ecosystem services and supporting anthropogenic activities. The lake basin supports a vibrant commercial horticulture and floriculture industry, whose growth has accelerated greatly in the past two decades due to the availability of sufficient freshwater for irrigation, good climatic conditions and existing links to local and international markets for vegetables and cut flowers. Further, the lake system supports tourism, fisheries, pastoralism and small holder subsistence food production systems. Irrigated horticulture occupies about 5025 ha around the lake (Legese Reta, 2011) cultivated by around 100 farms varying in size from ~1 ha to over 200ha (LNGG 2005; FBP 2012), while small scale farms averaging 2.5ha dot the entire basin, especially on its upper catchment. The growth of employment in the horticulture industry has triggered high annual population growth rates of 6.6% from 237,902 people in 1979 to 551,245 in 2009 (WWF 2011). This rapid population growth is responsible for the mushrooming of unplanned settlements around the lake and the problem of sewerage and solid waste disposal often associated with such settlements.

Besides water quality, expansion in agriculture has also had an impact on the quantity of water resources in the basin. Becht & Harper (2002) claim that water abstraction for irrigation has a measurable impact on the lake level. Their model shows a deviation of observed lake level from the simulated level since the onset of intensive flower industry around the lake in the early 1980's and estimated a drop in the long term average Lake level by 3-4 m as a result of abstractions.

Table 1: Estimates of the Lake Naivasha Water Balance[1] (million m^3 per year)

	McCann (1974)	Gaudet and Melack (1981)	Ase, Sernbo & Syren (1986)		Becht and Harper (2002)
Hydrologic budget item ($10^6 m^3 yr^{-1}$)	various sources and years	1973-1975 average (including Oloiden)	1972-1974*	1978-1980	1932-1981
Total inflow	380	337	279	375	311
Precipitation	132	103	106	135	94
River Discharge	248	234**	148	215	217
Total outflow	380	368	351	341	312
Evaporation from lake (including swamp)	346	312	284	288	256
Groundwater outflow (including abstractions)	34	56	67***	53***	56***

*For this study the numbers from the water level changes (in mm) to actual volumes have been recalculated, using the height-area relation presented by Åse et al. (1986, Figure 2.7). Two errors made in summations by Åse et al. (1986), Table 4.3 have been corrected. These are the values for July 1973 and April 1974.

**Including 'seepage in' from the northern section of the lake.

***Derived from the difference between the observed lake volume changes and the calculated volume changes as reported by Åse et al. (1986) and Becht and Harper (2002) respectively

Verschuren et al. (2000) demonstrate that, consistent with natural climate variability, Lake Naivasha has practically dried-up completely for decades and even centuries in the past. These trends are mainly driven by climate-related changes, especially the volatile rainfall patterns of semi-arid eastern Africa which have led to a substantial fluctuation in the lake's depth, volume and ecological characteristics in the past centuries. As indicated in Figure 2, water availability in the Lake Naivasha basin has been very unstable historically as a result of volatile weather conditions, where periods of average and above average rainfall alternate with prolonged drought. This condition has the implication that basin-wide institutions for water management will have difficulties to remain stable, as the surface inflows are highly volatile.

[1] Note that this balance does not account for subterranean recharge and back flows from irrigation.

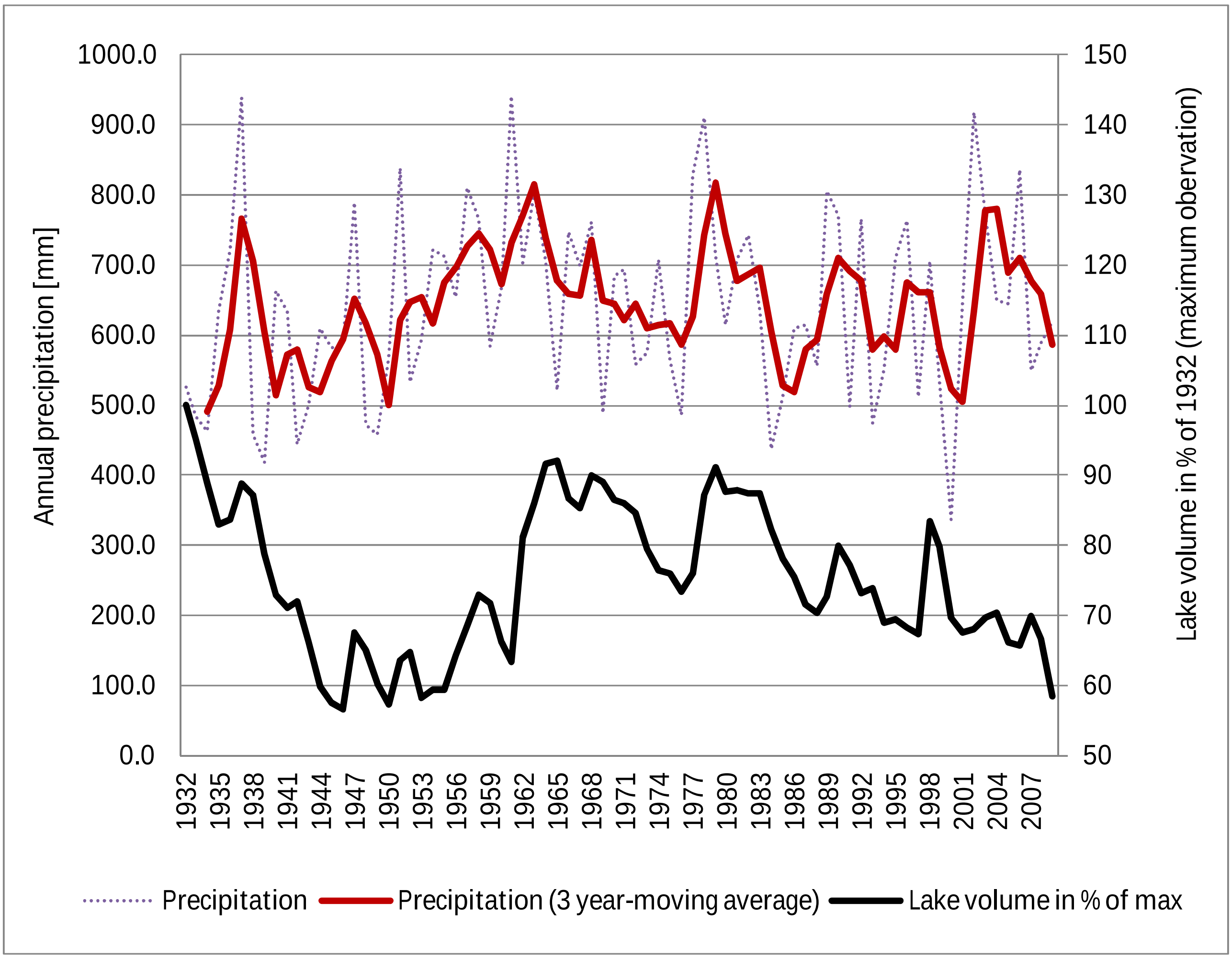

Figure 2: Trends in precipitation and Lake Naivasha volume (1932-2010)
Source: Legese Reta (2011).

The rapid growth of the flower industry, but also population growth (KNBS 2009) and expanding smallholder irrigation has increased the pressure on the volatile water resources of the Naivasha basin. Massive water use for irrigation in particular increases the likelihood that the lake may shrink or fall completely dry during drought periods. The social-ecological stability of the lake basin has changed, as dependence of livelihoods on water use has increased dramatically. But as the Lake Naivasha SES consists of numerous non-linear and interrelated hydrological, ecological, agronomic and economic processes, its resilience with respect to droughts or over-use of water is very difficult to assess intuitively. Systematic analyses based on numerical simulations offer the possibility to explore a) the impact of different water scarcity scenarios and b) the suitability of both existing and proposed water management institutions.

The Lake Naivasha Hydro-Economic Basin Model (LANA-HEBAMO), a numerical simulation model based on mathematical programming, was developed for this purpose and written in the numerical modelling software GAMS (see www.gams.com). In hydro-economic basin models, water use is principally driven by economic considerations, but under the hydrologic and other bio-physical constraints relevant for the basin in question. This technical documentation explains the model's spatial structure (section 2), its biophysical and agronomic features (section 3) and presents the results of some rainfall-related baseline scenarios (section 4). The annex contains the complete set of algebraic model equations.

2 The LANA-HEBAMO model

2.1 Spatial structure

The spatial and temporal structure of LANA-HEBAMO is set up in the same fashion as with most conventional Hydro-economic River Basin Models (HERBMs). It contains a GAMS set structure resembling a node-network of catchment areas, river reaches, reservoir, aquifers, and demand locations (figure 3).

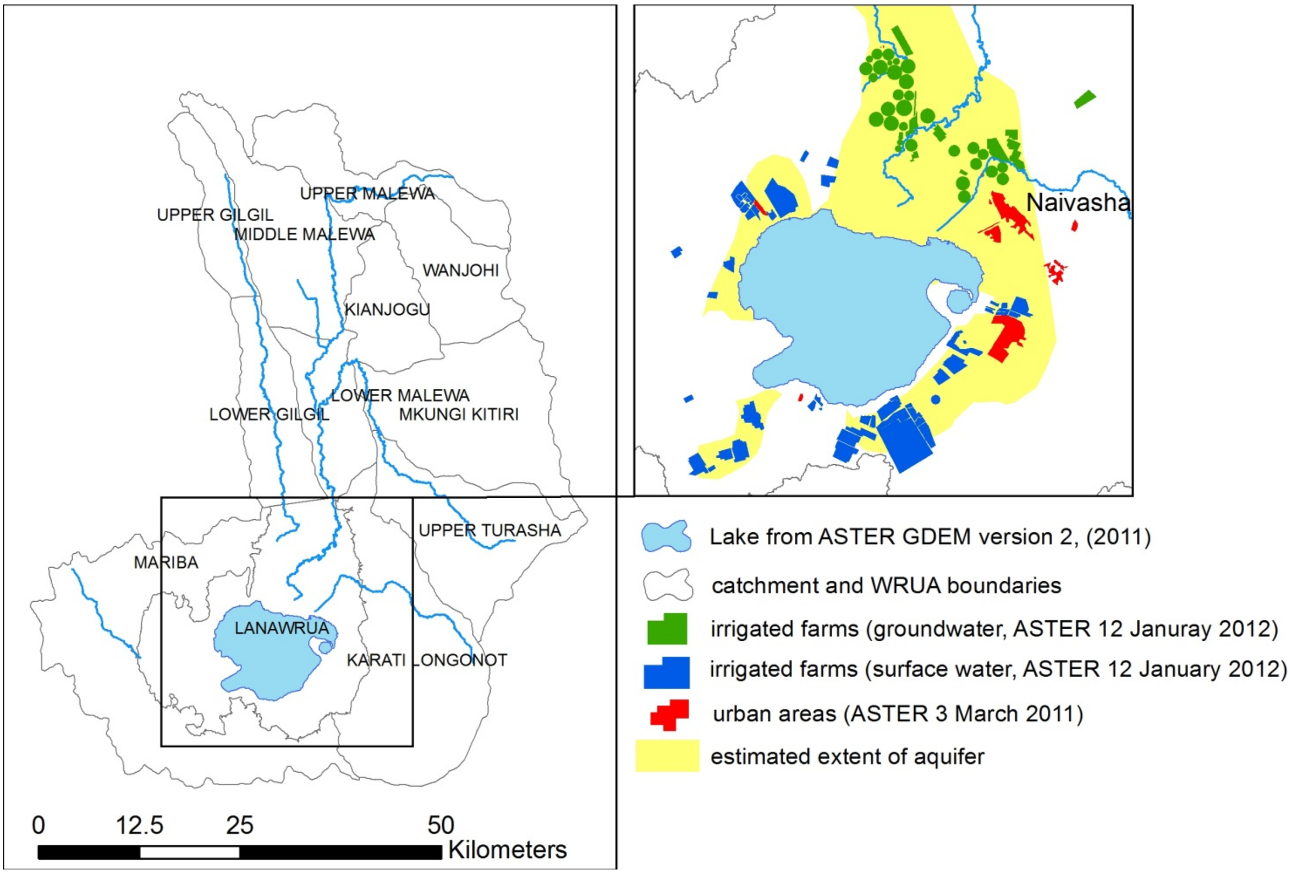

Figure 3: The Lake Naivasha Basin (left) and the Lake Naivasha area (right).

In the case of the Lake Naivasha catchment, the network is characterized by the lake being the terminal node that is fed through rivers. Rivers transport runoff from rainfall in the Naivasha catchment area. 'Nodes' with an area attached are the areas belonging to one of the twelve water resource user associations (WRUAs) which in sum cover the entire catchment. It is thus assumed that any renewable water resources in the basin are generated by runoff from rainfall on the WRUAs' areas. Runoff provides water to the rivers that then ultimately feed the Turasha Dam and Lake Naivasha. Groundwater use is assumed to happen in the lake area only, where a shallow aquifer is in hydraulic interaction with the Lake.

2.2 Biophysical and agronomic features

Climate and water supply

The climate in the lake Naivasha basin is not homogenous across locations. In the lake area, semi-arid conditions dominate, while cooler, but humid conditions can be found upstream in higher altitude. Figure 4 displays monthly levels of temperatures (red), rainfall (blue) and evaporation (black) in Naivasha (lake area) and South Kinangop (upper Naivasha catchment).

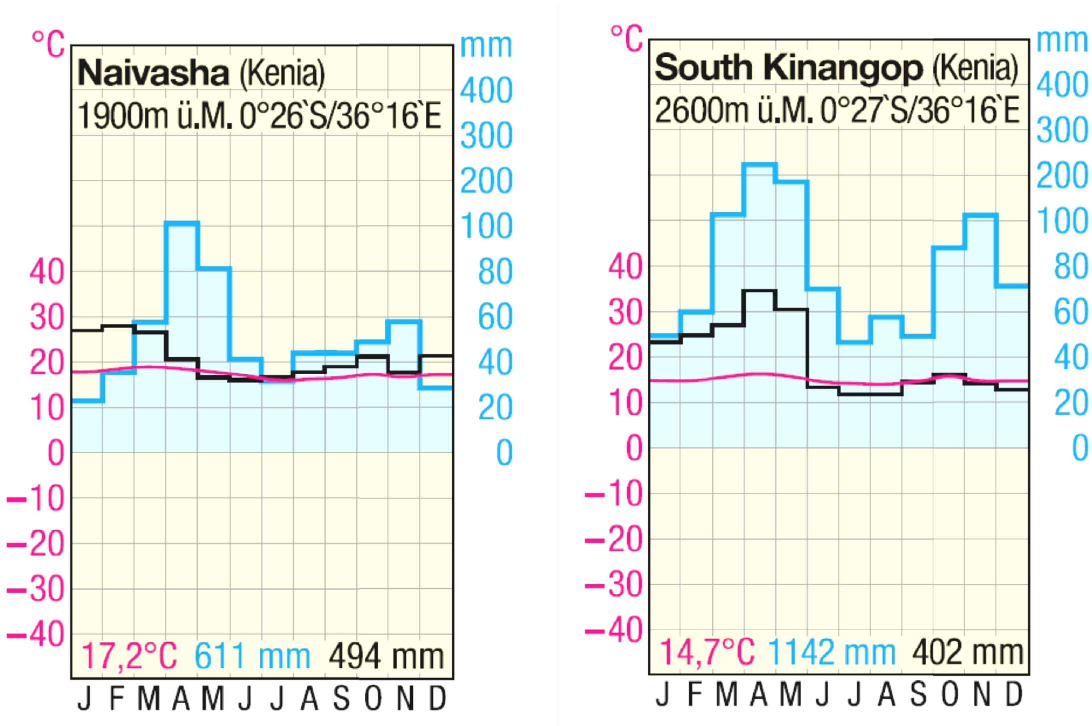

Figure 4: Climate charts for Naivasha and the Kinangop plateau
Source: Stein (2009)

Influenced by large differences in altitude the climate in the Lake Naivasha basin is spatially very diverse with annual precipitation averages ranging from ~650mm around Lake Naivasha, up to ~1300mm in the mountain forests of the Aberdares. Precipitation distribution is typically bi-modal with rainy seasons in the periods March-May and October-November. The average annual pan evaporation rate (measured at the Naivasha DO station) is 1790 mm, contributing to a semi-arid climate in the lake area. Evaporation in higher altitudes is somewhat lower. Mean monthly minimum temperatures at Lake Naivasha range from 6 $°C$ to 10$°C$, while mean monthly maximum temperatures range from 26 $^{\circ}C$ to 31$^{\circ}C$. Average monthly temperatures range from 15.9 $^{\circ}C$ to 17.8 $^{\circ}C$ (De Jong, 2011).

Water availability in LANA-HEBAMO is driven by monthly rainfall which is interpolated to the WRUA areas. A rainfall dataset from the Kenyan Meteorological Department (KMD) is used to estimate rainfall series for each of the catchments (WRUA catchments) for the period 1957-2010. The dataset contains daily rainfall records for 67 stations inside and directly around the Lake Naivasha basin. The KMD database is complemented with some of the other data collected in the field or obtained from the Water Resources Management Authority (WRMA) offices in Nakuru and Naivasha, Kenya. Taking into consideration a daily rainfall threshold of <250mm (Bärring 1988), 9 values were identified as outliers and therefore omitted from the dataset. Spatial interpolation of daily rainfall records is done using squared inverse distance interpolation. Spatial interpolation was only applied for cases where three or more rain gauges with records were available. In other cases a weather generator based on Neitsch et al. (2011) was used. The stations that have been interpolated are distributed over the Lake Naivasha basin. To estimate representative values for daily rainfall in sub-catchments, artificial stations have been created on the centroids of the sub-catchments using inverse distance interpolation.

Monthly rainfall generates water supply in two ways. First, rainfall enters the crop field balance of rainfed crop or crops where supplementary irrigation is possible. Secondly, rainfall on the level of the 13 sub-basins generates surface runoff that feeds the river system of the basin. A simple statistical relation between the actual rainfall and the produced runoff has been implemented in the model (equation (10)) producing a variable monthly runoff coefficient. The runoff calibration model estimates the coefficients of the runoff power function by running the hydrological sub-model simultaneously across the calibration period (1960-1985, fully dynamic model) while minimizing the squared difference between observed and estimated lake levels in equation (21). The calibration model consists of equations (10) to (21), but with the years of the calibration period as an additional dimension. The calibration period was chosen to end in 1985 in order to minimize the influence of water abstraction for irrigation and other use on the hydrological balances.

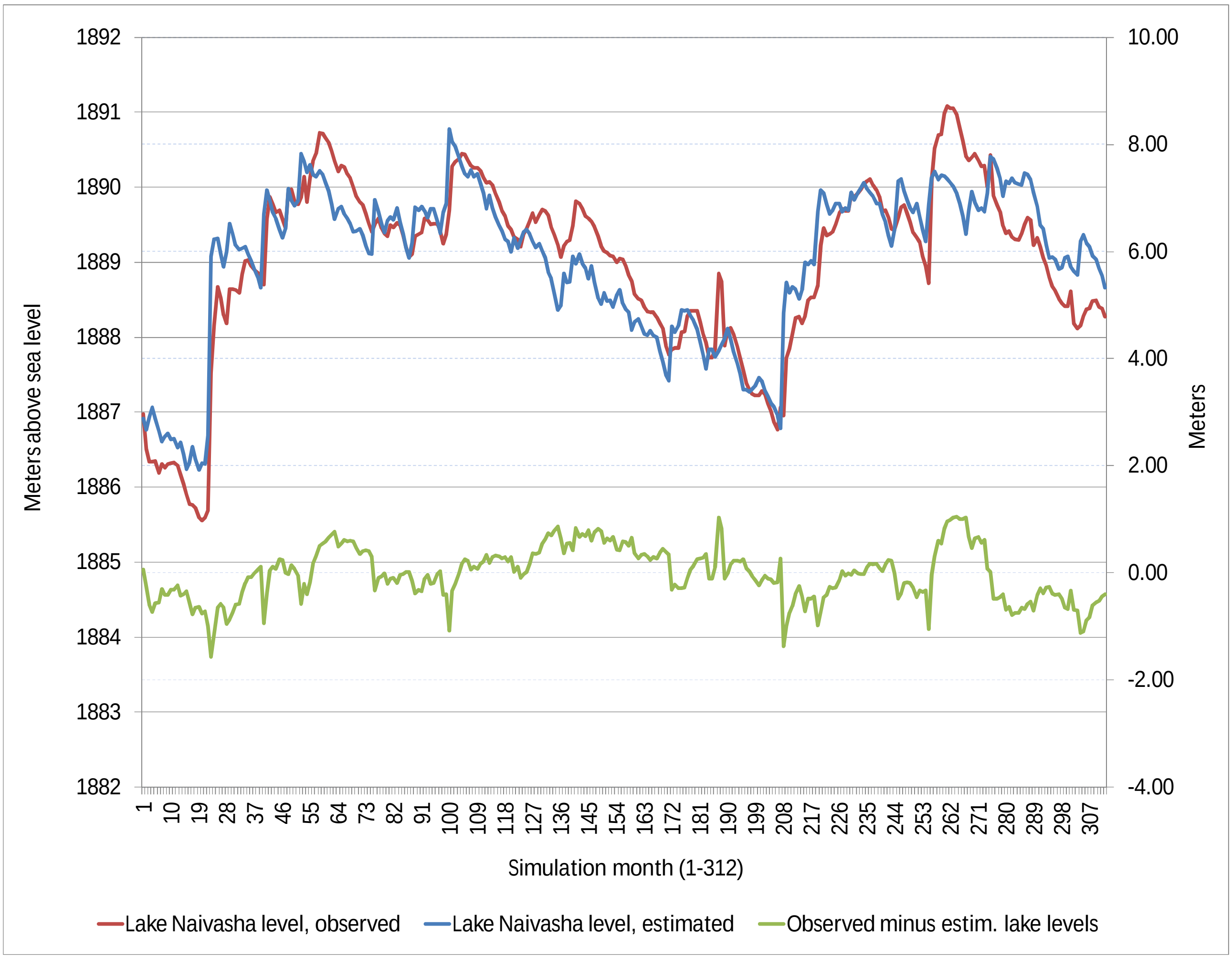

Figure 5: Calibration of runoff – observed versus estimated levels of Lake Naivasha, Jan 1960 – Dec 1985

Figure 5 illustrates the runoff calibration result by contrasting observed and estimated lake levels. Observed and estimated lake levels correlate with 0.907. The calibration resulted in the following runoff equation:

$$\text{Runoff [mm]} = 0.000443 \cdot \text{Rainfall [mm]}^{2.081}$$

To validate the hydrologic sub-model, the above runoff function was applied to the recursive LANA-HEBAMO. The validation run comprises the period from January 1995 – December 2009. The result of the unadjusted validation run is shown in figure 6 (red line). The validation runs now also contain water use for irrigation by the horticultural industry that started in the 1980s. As compared to observed lake levels (green line), it is striking to see that the unadjusted runoff function produces lake levels that are systematically too low, and that correlation with observed values is down to 0.69. One reason could be that runoff as a share of rainfall might have increased in recent decades as a consequence of cropland expansion and deforestation in the upper catchment of Lake Naivasha, which suggests that land use and cover change (LUCC) should be part of an improved runoff estimation effort. But as there are no time series on LUCC available, a preliminary fix of this problem is to generally adjust simulated runoff by a factor of 1.22, which leads to a much better fit (blue line) and correlation with observed lake levels (0.82). Given limited data availability, we believe that this runoff model is useful to produce plausible analyses of water availability in the vicinity of Lake Naivasha under different rainfall scenarios.

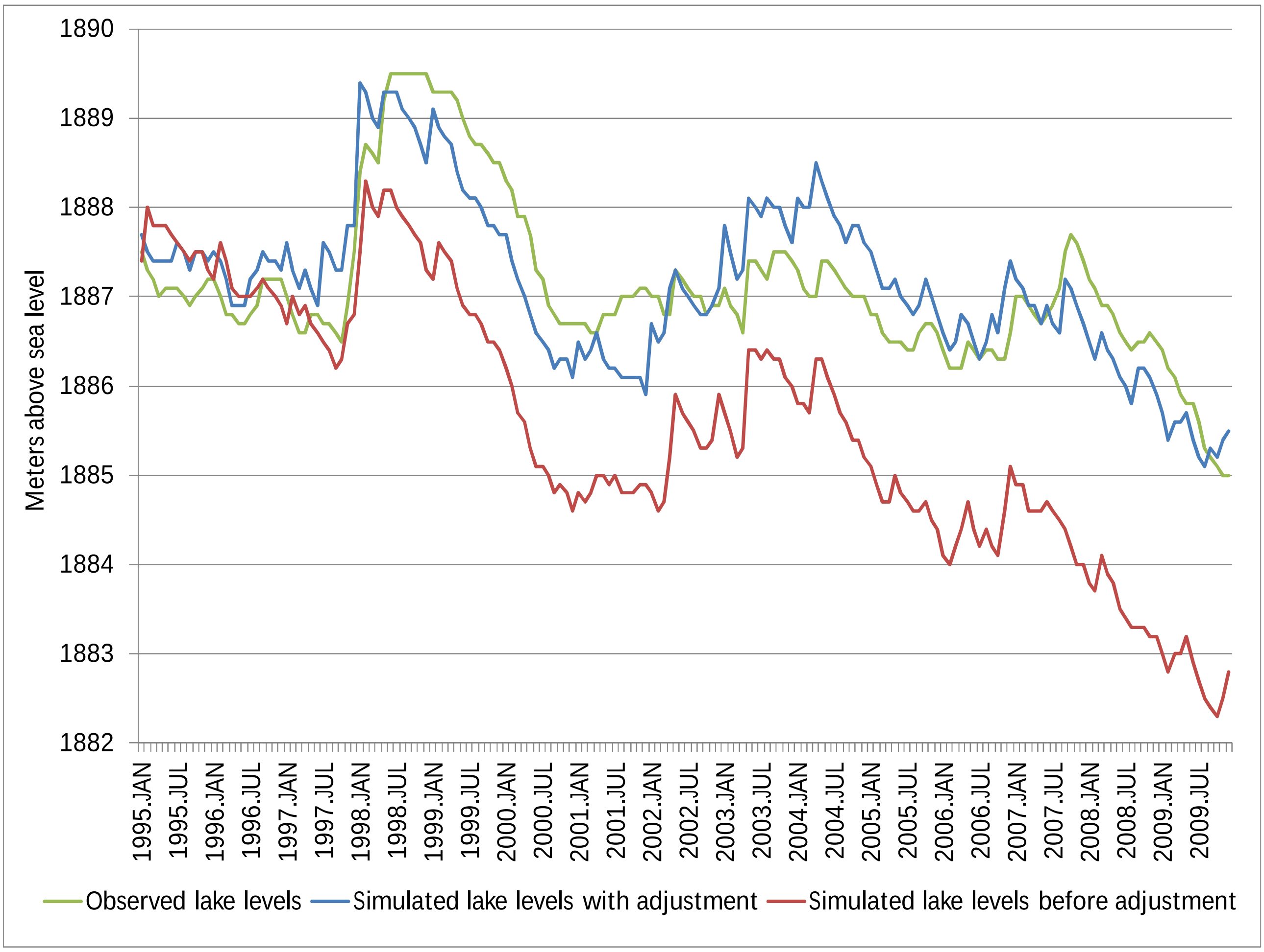

Figure 6: Validation of the runoff equation– observed versus simulated levels of Lake Naivasha, Jan 1995 – Dec 2009

Water demand

Crop cultivation in the Naivasha basin is characterized by a pronounced dichotomy between the upper Naivasha catchment and the lake's riparian areas. In the upper catchment, small-scale farmers mainly cultivate subsistence crops such as maize, potatoes and peas, supplemented by some commercial growing of vegetables such as French beans and carrots. In the wider lake area, ownership structures are completely different. Since colonial times, large scale farms own the vast majority of arable land. Since the 1980s, a horticulture industry (vegetables and cut flowers) that relies heavily on irrigation has grown steadily in the Lake area. In the upper catchment, by contrast, most crops are grown in rainfed agriculture, as the climate is more humid and irrigation infrastructure mostly lacking.

The baseline of the model focuses on current irrigated crop areas. For the lake region, Mekonnen and Hoekstra (2010) report 4450 ha of irrigated crops, meaning that 100% of crop area in the lake region is irrigated. Of these, 1190 ha are roses and other flowers grown in greenhouses. Mpusia (2006:61) cites the estimates of various previous studies and arrives at 5400 ha of irrigated area in the entire Naivasha basin (including the catchment), of which greenhouses cover are 1600 ha. Unfortunately he does not clarify how these areas are distributed across sub-catchments, so a preliminary solution is to allocate all these areas to the LANAWRUA region. Within the LANAWRUA area, Musota (2008:48) distinguishes between a North and a South Lake area, a distinction which has also been adopted by the LANA-HEBAMO model, as these two areas are quite distinct regarding crop mix and sources of irrigation water. The figures mentioned by the latter three publications were used to set up the database of the model baseline.[2]

In the current model version, only irrigated crop areas influence the basin water cycle. Irrigated crops (indoor roses, outdoor flowers, irrigated vegetables and fodder) are assumed to be supplementary irrigated to achieve maximum yields. Estimates for the amount of irrigation applied (minus return flows) are taken from Mpusia (2006) from a fieldwork period of a series of days in September 2005. Actual average evapotranspiration was determined to be 3.5mm for the irrigation of flowers inside greenhouses and 5.4mm for outdoor irrigation. When we take into account annual average rainfall (695mm), the additional crop water requirement is around 3.5mm (net) as well. The amount of 3.5mm is well below the daily applied amount of irrigation of around 5.0mm. Data on actual amounts of monthly water abstractions originate from two sources:

- A monthly abstraction data-set from the Lake Naivasha Growers Group (LNGG) for January 2003 – December 2005 indicates that 5.0mm is applied (LNGG 2005; Musota 2008). LNGG represents a group of 28 farmers in 2005, jointly irrigating 922ha in the same year.
- A monthly abstraction data-set from the Flower Business Park for March 2008 – April 2012 indicates that 4.9mm is applied (FBP, 2012).

It is assumed that all of the excess irrigation (above 3.5mm/day) is returned to the lake and the lake aquifer. Therefore the irrigation amount minus return flows assumed in this study is 3.5mm/day. For greenhouse roses, irrigation thus provides 152 mm of water per month regardless of rain, while other crops' water requirements are a function of potential evapotranspiration (ET0), and crop- and stage-specific Kc-values.[3] Irrigated outdoor crops receive water from rain and supplementary

[2] More recent estimates by ITC researchers have not yet been published.

[3] This method is relatively crude, as it does not sufficiently consider the specific local climate and soils, soil fertility and management, and local crop varieties. In the longer planned to introduce crop-water functions derived from

irrigation to meet crop-specific water requirements of maximum yields. Gross water demand for irrigation adds up to roughly 75 million cubic meters per annum in the basin, of which 19 million cubic meters are return flows at an irrigation efficiency of 25%. In addition, non-irrigation water demand – consisting of household water demand within the basin plus water transfers outside the basin to the town of Nakuru – are estimated at 15 million cubic meters annually. These non-irrigation water demands are assumed to be exogenous in the current model version, and adjusted from one simulation year to the next along with population growth.

Decision variables in the model

LANA-HEBAMO in its current version is based on the assumption of basin-wide aggregate optimization. This means that productive resources are allocated among locations, time periods and irrigable crops such that the sum of the profits of all water users in the basin is maximized. It is important to realize that this aggregate optimization format has an institutional implication: it reflects a situation of either a) central planning of land and water use, or b) assumes the existence of perfectly functioning markets for water use rights (Kuhn and Britz 2012). Both assumptions are not realistic in the case of the Naivasha basin where neither central planning nor water trading exist. The result of aggregate optimization is therefore bound to deviate from a reality which is characterized by an absence of basin-wide water management, but rather represents a best-case scenario with a benevolent central planner in the background, an assumption on which the interpretation of the baseline scenarios in the next section will rest. The decision variables that can be altered to arrive at this basin-wide maximum involve land and water use, the latter partly coupled to land use when it comes to decisions on irrigated crop areas. Land use involves both irrigated and non-irrigated crop areas in the individual WRUAs which are assumed to be aggregate farming decision units. The major decisions on water use are made implicitly by deciding on the acreage of irrigable[4] crops (flowers, vegetables, and some fodder). The acreage of an irrigable crop is determined by overall water availability and the specific profitability of the crop as compared to other crops. Once area is determined, crop water demand is calculated as the difference between rainfall (zero in the case of greenhouse crops) and total crop water demand due to maximum ET. If monthly rainfall is higher than crop ET, an excess runoff variable was introduced to capture this imbalance.

---

adapted AQUACROP simulations (see http://www.fao.org/nr/water/aquacrop.html). These would allow to calculate weather-dependent crop yields in both irrigated and rainfed agriculture, and, based on this, analyse incentives to expand supplementary irrigation in the Upper Catchment.

[4] For the construction of the model baseline it is assumed that crops for which no irrigation can currently be observed are non-irrigable crops, an assumption that may be relaxed in scenarios on future water use in the basin.

While water use for individual crops is a function of crop area, crop water demand and rainfall, two other decisions can be made by the central planner. First, how much water will be allocated to which WRUA, a decision which is closely linked to crop areas within WRUAs, and, second, the source of irrigation water. Here it is assumed that local choices are limited: in the WRUAs of the upper catchment, water for irrigation is assumed to be available from river reaches flowing through the WRUA's area. Theoretically, farmers could use water from the Turasha dam and other small reservoirs, but the necessary infrastructure was not observed during the surveys for data collection. Turasha is currently used to smooth water supply to the town of Nakuru outside the Naivasha basin. On the other hand, the WRUA in the Lake area (LANAWRUA) is assumed to have no access to river water, but can choose between lake water and groundwater. Groundwater may be more expensive to pump, as groundwater levels are lower than lake levels, but groundwater may have the advantage to be less easy to control by the WRMA (Water Resource Management Agency), the public body which is mandated to allocate water use permits and collect charges for water (WRMA 2010).

3 Illustrative scenarios

This section presents a couple of basic scenarios that illustrate the behavior of the simulation model under different assumptions on water availability. First, lake balances for three different rainfall situations in the basin are presented. As indicated in table 2, water abstractions play an important role in determining the lake water balance.

Table 2: Lake Naivasha Water Balances under average (μ), wet ($\mu+\sigma$) and dry rainfall conditions ($\mu-\sigma$)[5]. Results in million m^3 per year, μ denotes the arithmetic mean, σ is one standard deviation.

	$\mu+\sigma$	μ	$\mu-\sigma$
Surface water inflows	454.2	176.4	36.4
Rainfall	207.3	90.5	0
Evaporation at 1887.5 masl	285.2	249.9	224.1
Abstraction (2010 estimate)	28.7	34.7	36.9
Subterranean discharge to the 'Lake Aquifer'	56.6	30.8	-7.7
Net Gain/Loss	290.9	-48.5	-232.4
Gain/loss in % of volume at 1887.5 masl (660 mio cbm)	44.1	-7.4	-35.2

Due to its low volume, the lake's level is highly sensitive to shifts in natural conditions (rainfall, surface inflow, evaporation and subterranean discharge) and human abstractions (irrigation and

[5] Note that this balance does not account for subterranean recharge and back flows from irrigation.

domestic use). The impact of water abstraction is likely to be felt more during periods of low surface inflows due to low rainfall. In a single dry year, the lake may lose 35% of its average volume with abstractions and 30% without abstractions. During the wettest years however, the lake would gain considerably, with or without abstractions.

Next, a simulation run over a decade of average conditions is presented in figure 7. This simulation can be interpreted as a test of the mid-term quantitative sustainability of water use. The main result is that the lake balance is negative throughout the simulation period, which means that the lake would shrink under average conditions and current water use patterns. However, the pace of decrease slows down considerably with decreasing lake volume. The reason is that the smaller lake surface allows for fewer evaporation losses. Evaporation of water from the lake surface is by far the most important loss factor. Water abstraction for irrigation accounts for only 13% of total outflows. This result supports similar findings in the scenarios of Becht & Harper (2002).

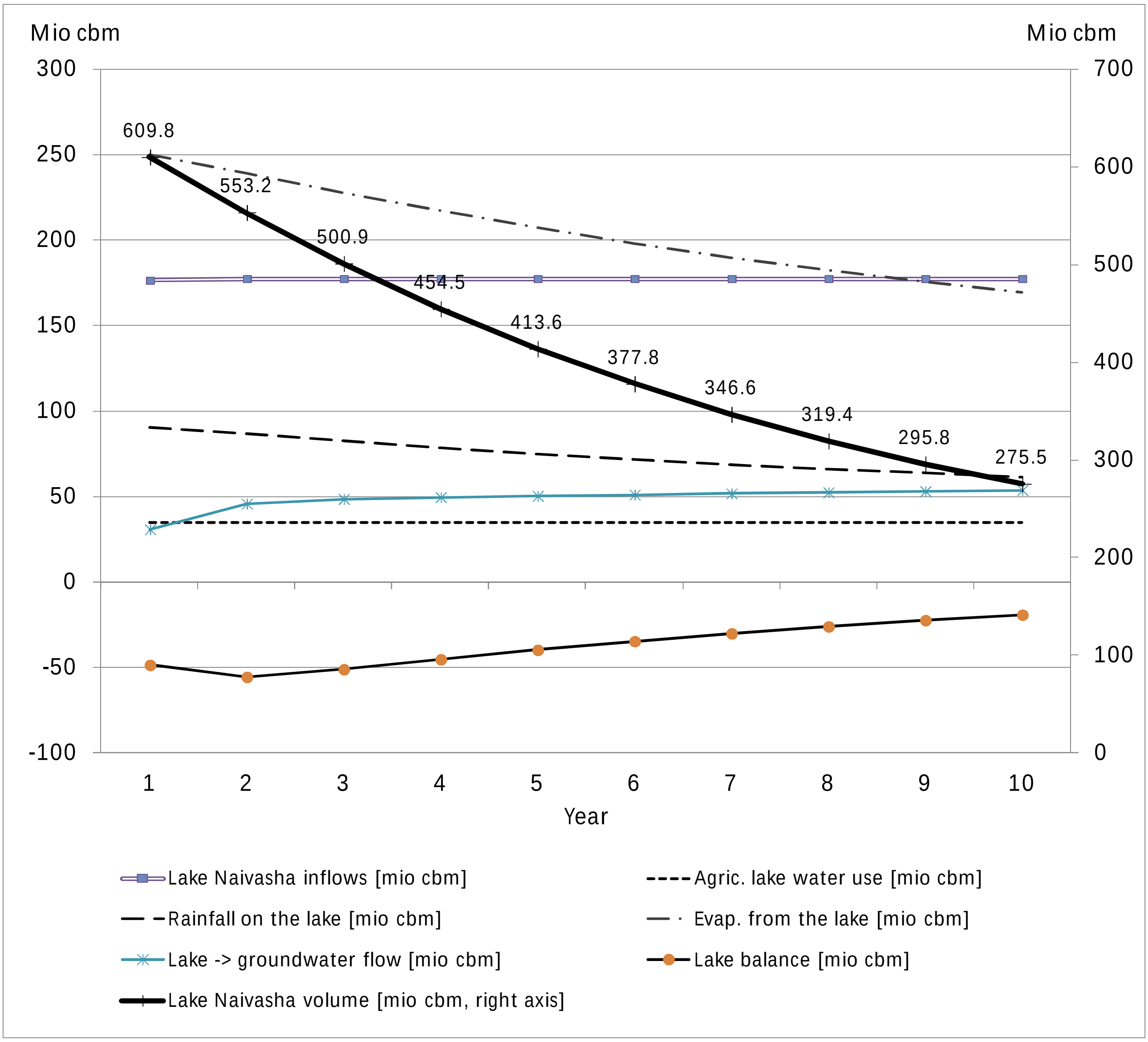

Figure 7: A 10-year model run under constant, average local rainfall conditions

4 References

Bärring, L. (1988). Reginalization of daily rainfall in Kenya by means of common factor analysis. *Journal of Climatology* 8(4), 371-389. doi: 10.1002/joc.3370080405

Becht, R. and D. M. Harper (2002). Towards an understanding of human impact upon the hydrology of Lake Naivasha, Kenya. *Hydrobiologia* 488(1-3): 1-11.

De Jong, T. (2011). *Water abstraction survey in Lake Naivasha basin, Kenya*. Wageningen University. BSc. Internship thesis.Wageningen. ftp://ftp.itc.nl/pub/naivasha/DeJong2011.pdf

FBP (2012). *Monthly abstractions and area under irrigation March 2008 - April 2012*. James Waweru - General Manager of Flower Business Park Management Limited. Naivasha, Kenya.

Harper, D. and K. Mavuti (2004). Lake Naivasha, Kenya: Ecohydrology to guide the management of a tropical protected area. *Ecohydrology and Hydrobiology* 4(3): 287-305.

KNBS (2009). *Population and Housing Census*. Nairobi, Kenya, Kenya National Bureau of Statistics.

Kuhn A., Britz W. (2012): Can hydro-economic river basin models simulate water shadow prices under asymmetric access? *Water Science & Technology* 66(4): 879-886. doi:10.2166/wst.2012.251

Legese Reta, G. (2011). *Groundwater and lake water balance of lake Naivasha using 3 - D transient groundwater model*. University of Twente Faculty of Geo-Information and Earth Observation ITC. thesis. Enschede, The Netherlands. ftp://ftp.itc.nl/pub/naivasha/ITC/LegeseReta2011.pdf

LNGG (2005). *Monthly abstractions and area under irrigation January 2003 - December 2005*. Lake Naivasha Growers Group. Naivasha, Kenya

Mekonnen, M. M. and A. Y. Hoekstra (2010). *Mitigating the water footprint of export cut flowers from the Lake Naivasha Basin, Kenya*. UNESCO-IHE. Delft, The Netherlands. ftp://ftp.itc.nl/pub/naivasha/PolicyNGO/Mekonnen2010.pdf

Mpusia, P. T. O. (2006). Comparison *of water consumption between greenhouse and outdoor cultivation*. ITC. MSc thesis. Enschede, The Netherlands. ftp://ftp.itc.nl/pub/naivasha/ITC/Mpusia2006.pdf

Musota, R. (2008). *Using weap and scenrios to assess sustainability water resources in a basin.case study for lake Naivasha catchment, Kenya.* ITC. MSc thesis. Enschede, The Netherlands. ftp://ftp.itc.nl/pub/naivasha/ITC/Musota2008.pdf

Neitsch, S. L., Arnold, J. G., Kiniry, J. R., & Williams, J. R. (2011). Soil and Water Assessment Tool Theoretical Documentation Version 2009: Texas Water Resources Institute.

Stein, C. (2009). Räumliche und klimatische Einordnung von Naivasha in Kenia. *Diercke 360°* 1/2009, www.diercke.de/bilder/omeda/Copy_Naivasha.pdf

Verschuren, D., K. R. Laird and B. F. Cumming (2000). Rainfall and drought in equatorial east Africa during the past 1,100 years. *Nature* 403(6768): 410-414.

WRMA (2010). *Water Allocation Plan - Naivasha basin 2010 - 2012*. Naivasha, Kenya, ftp://ftp.itc.nl/pub/naivasha/PolicyNGO/WRMA2010.pdf

WWF (2011). Seeking a sustainable future for Lake Naivasha. WWF Report 2011, prepared by PegaSys Strategy and Development. ftp://ftp.itc.nl/pub/naivasha/PolicyNGO/WWF2011.pdf

5 Annex: Model equations

(1) Objective function

$$\max\ V\_GOALVAR = \sum_{dma} VAGPROFIT_{dma}$$

(2) Agricultural profits

$$VAGPROFIT_{dma} = \sum_{crop}\left(VCROPAREA_{dma,crop} \cdot \left(\begin{array}{l} PCROPPRIC_{crop} \cdot VCROPYIEL_{dma,crop} \\ -\sum_{prof}\left(PFACTNEED_{crop,prof} \cdot PFACTPRIC_{crop,prof}\right) \end{array} \right)\right)$$
$$-VPMP\_COST_{dma} - \sum_{gw}\sum_{pd}\left(VPUMP\_DMA_{gw,dma,pd} \cdot PGW\_PRICA_{dma}\right)$$
$$-\sum_{n}\sum_{pd}\left(VFL\_N\_\_DMA_{n,dma,pd} \cdot PSW\_PRICA_{dma}\right) - \sum_{res}\sum_{pd}\left(VFLRES\_DMA_{res,dma,pd} \cdot PRS\_PRICA_{dma}\right)$$

(3) PMP cost term

$$VPMP\_COST_{dma} = \sum_{crop}^{n} PMPA_{dma,crop} \cdot VCROPAREA_{dma,crop} + PMPB_{dma,crop} \cdot \left(VCROPAREA_{dma,crop}\right)^2$$

1. Yield formation as a function of rain and irrigation water application

(4) ET from rainfall

$$VETA\_RAIN_{crop,dma} = \frac{PETA^{max}_{crop,dma} \cdot PETA^{min}_{crop,dma} \cdot \exp\left(PETA\_R_{crop,dma} \cdot \sum_{pd} PEFF\_RAIN_{crop,dma,pd}\right)}{PETA^{max}_{crop,dma} + PETA^{min}_{crop,dma} \cdot \left(\exp\left(PETA\_R_{crop,dma} \cdot \sum_{pd} PEFF\_RAIN_{crop,dma,pd}\right) - 1\right)}$$

(5) Total monthly ET

$$VETA\_STAG_{dma,crop,pd} = VWATUSEHA_{dma,crop,pd} + \begin{cases} PWATREQCR_{crop,pd} & \forall\ crop \neq crsi \\ \left(\dfrac{VETA\_RAIN_{dma,crop}}{100}\right) \cdot PRAINDSTR_{dma,crop,pd} & \forall\ crop = crsi \end{cases}$$

(6) Seasonal ET

$$VETA\_SEAS_{dma,crop} = \frac{VETA\_STAG_{dma,crop,pd}}{PWATRQFCT_{dma,crop,pd}}$$

(7) Yield function[6]

$$VCROPYIEL_{dma,crop} = P\_Y^{\max}_{crop} \cdot \frac{100 \cdot \dfrac{P\_Y^{min}_{dma,crop}}{P\_Y^{max}_{dma,crop}} \cdot \exp\left(PY\_R_{dma,crop} \cdot VETA\_SEAS_{dma,crop}\right)}{100 + \dfrac{P\_Y^{min}_{dma,crop}}{P\_Y^{max}_{dma,crop}} \cdot \left(\exp\left(PY\_R_{dma,crop} \cdot VETA\_SEAS_{dma,crop}\right) - 1\right)}$$

[6] In the current model version, crop yields are fixed, and this equation is inactive.

2. Hydrologic processes which link water sources with irrigation water use

(8) Source of irrigation water

$$\sum_{crop} V\_\_W\_A\_CR_{dma,crop,pd} = VFL\_N\_DMA_{n,dma,pd} + VPUMP_{gw,dma,pd}$$
$$+VFLRESDMA_{res,dma,pd} \text{ (if } n, gw, res \text{ match } dma\text{)}$$

(9) Water use per hectare and total

$$VWATUSEHA_{dma,crop,pd} = \frac{V\_W\_CR\_A_{dma,crop,pd}}{VCROPAREA_{dma,crop}}$$

3. Hydrologic equations for river nodes, groundwater, reservoirs and Lake Naivasha

(10) Runoff from rainfall in mm (power function)

$$VRAIN\_RUN_{dma} = P\_BETA^{I} \cdot PTOT\_RAIN_{dma,pd}{}^{P\_BETA^{II}} \text{ (runoff calibration model)}$$

$$VRAIN\_RUN_{dma} = P\_BETA^{I} \cdot PTOT\_RAIN_{dma,pd}{}^{P\_BETA^{II}} \cdot P\_ALPHA \text{ (validation and simulation models)}$$

(11) Local runoff into the river node of a in sub-basin (WRUA area)

$$VLOCRUNOF_{n,pd} = \frac{PTOT\_RAIN_{dma,pd}}{100} \cdot VRAIN\_RUN_{dma} \cdot PTOT\_AREA_{dma} \text{ (when } dma \text{ matches } n\text{)}$$

(12) River node balance

$$VRIVERFLO_{n,n\_lo,pd} + VFL\_N\_RES_{n\_res,pd} + VFL\_N\_DMA_{n\_dma,pd}$$
$$= VRIVERFLO_{n\_up,n,pd} + VLOCRUNOF_{n,pd} + VFL\_RES\_N_{res\_n,pd}$$

(13) Intertemp. groundwater heads

$$V\_GW\_HEAD_{gw,pd} = V\_GW\_HEAD_{gw,pd-1} + VGWCHANGE_{gw,pd}$$

(14) Groundwater change balance

$$VGWCHANGE_{gw,pd} \cdot PGW\_YIELD_{gw} \cdot P\_GW\_AREA_{gw} \cdot 10$$
$$= VRESDISCH_{res,gw,pd} - VPUMP\_DMM_{gw,dmm,pd} - VPUMP\_DMA_{gw,dma,pd} - P\_DISCHRG_{gw,pd}$$

(15) Aquifer recharge/discharge

$$VRESDISCH_{res,gw,pd} = P\_CONDUCT_{res,gw} \cdot \left(VRES\_LEVL_{res,pd} - V\_GW\_HEAD_{gw,pd}\right)$$

(16) Lake or reservoir balance

$$VSTOR\_RES_{res,pd} = VSTOR\_RES_{res,pd-1} + VFL\_N\_RES_{n,res,pd} + VRES\_PREC_{res,pd} - VFLRESDMM_{res,dmm,pd} - VFLRESDMA_{res,dma,pd} - VFL\_RES\_N_{res,n,pd} - VRES\_EVAP_{res,pd} - VRESDISCH_{res,gw,pd}$$

(17) Lake area = f(Lake volume)

$$VRES\_AREA_{res,pd} = PRESAREAB \cdot VSTOR\_RES_{res,pd}{}^{PRESAREAP} \cdot 100$$

(18) Lake level = f(Lake area)

$$VRES\_LEVL_{res,pd} = PRESLEVLB \cdot (\frac{VRES\_AREA_{res,pd}}{100}) + PRESLEVLC$$

(19) Rainfall on the lake

$$VRES\_PREC_{res,pd} = VRES\_AREA_{res,pd} \cdot \frac{PRES\_PREC_{res,pd}}{100}$$

(20) Evaporation from the lake

$$VRES\_EVAP_{res,pd} = VRES\_AREA_{res,pd} \cdot \frac{PRES\_EVAP_{res,pd}}{100}$$

(21) Objective function of the runoff calibration model[7]

$$VMINSQDEV = \sum_{pd,year} \left(PLAKELEVEL_{pd,year} - VRES\_LEVL_{'lake',pd,year} \right)^2$$

4. Fixed water demand for non-agricultural water use

(22) Withdrawals by municipal demand sites

$$VINFLOW\_M_{dmm,pd} = VFLRESDMM_{res,dmm,pd} + VPUMP\_DMM_{dmm,pd}$$

Model indices (sets)

year	Years of calibration, validation or simulation periods
pd	Time index within a year (months)
dma	Oasis (irrigation water demand site)
dmm	Municipal demand site
n	River node where water is withdrawn
n_lo	River node located downstream of the actual node

[7] The runoff calibration model estimates the coefficients of the runoff power function by running the hydrological sub-model simultaneously across the calibration period (1960-1985) while minimizing the squared difference between observed and estimated lake levels in equation (21). The calibration model consists of equations (10) to (21), but with the years of the calibration period as an additional dimension.

n_up	River node located upstream of the actual node
gw	Groundwater aquifers belonging to oasis *dma*
res	Reservoir
crop	Crop, rainfed or irrigated
crpf	Irrigated crop with yield fixed to maximum yield (ETmax = rain + irrigation)
crsi	Irrigated crop with variable yield dependent on water application (ETact = rain + irrigation)
prof	Production factors (labour, machinery, fertilizer, pesticides)

Model variables

V__W_A_CR	Irrigation water available to a crop both from surface water and groundwater
V_GOALVAR	Objective variable (total water-related benefits in the basin)
V_GW_HEAD	Groundwater table of an aquifer
VAGPROFIT	Gross profit of farmers in oasis *dma*
VCROPAREA	Crop area for a crop per oasis
VCROPYIEL	Crop yield in tons
VETA_RAIN	Seasonal (annual) crop evapotranspiration due to rainfall only
VETA_SEAS	Seasonal (annual) total crop evapotranspiration (*ETa*)
VETA_STAG	Stage (monthly) crop evapotranspiration (*ETa*)
VFL_N_DMA	Water abstraction for irrigation from a river
VFL_N_RES	Flow from a river reach to a reservoir
VFL_RES_N	Flow from a reservoir to the river
VFLRESDMA	Water withdrawal from the reservoir for irrigation
VFLRESDMM	Water withdrawal from the reservoir for municipal demand sites
VGWCHANGE	Change in the groundwater table per aquifer and period
VINFLOW_A	Available river water for a demand site *dma*
VLEACHFCT	Leaching factor
VLOCRUNOF	Local runoff from total rainfall
VPMP_COST	Nonlinear cost term to calibrate crop areas (PMP = Positive Mathematical Programming)
VPUMP_DMA	Amount of pumped groundwater for irrigation purposes
VRAIN_RUN	Local runoff generated by rainfall [mm]
VRES_AREA	Surface area of the lake
VRES_EVAP	Evaporation per month from the lake
VRES_LEVL	Fill level of the lake
VRES_PREC	Rainfall per month on the lake
VRESDISCH	Flows between lake and adjacent groundwater aquifer
VRIVERFLO	Water flow from an upstream river node to a downstream river node
VSTOR_RES	Reservoir storage

Model coefficients (parameters)

PMPA	Constant in PMP cost term
PMPB	Slope parameter in PMP cost term
P_ALPHA	Parameter used to adjust calibrated runoff to the runoff in the validation period
P_CONDUCT	Subterranean flow between lake and aquifer at 1m level difference
P_DISCHRG	Fixed subsurface discharge of groundwater from the basin
P_GW_AREA	Surface of the groundwater aquifer
PCROPPIRC	Selling price of the crop
PEFF_RAIN	Effective rainfall in mm
PETA_MIN	Minimum ET in the logistic ET approximation function
PETA_MAX	Maximum ET in the logistic ET approximation function
PETA_R	Slope coefficient of the logistic ET approximation function
PFACTNEED	Non-water production factor needs (fertiliser, labour etc.)
PFACTPRIC	Production factor prices
PGW_PRICA	Costs for using groundwater
PGW_YIELD	Groundwater yield coefficient
PSW_PRICA	Costs for using surface water
PRS_PRICA	Costs for using reservoir water
PINFLOW_M	Water use of households and industry (fixed, shifted between simulation years)
PIRR_EFFY	Irrigation efficiency factor (constant)
PMAXYIELD	Maximum yield for the different crops (per ha)
PRAINDSTR	Distribution of effective rainfall across the months of the growing period of a crop
PRES_EVAP	Evaporation losses from the reservoir
PRESLEVLC	Constant parameter in the lake level approximation function
PRESLEVLB	Slope parameter in the lake level approximation function
PRESAREAB	Slope parameter in the lake area approximation function
PRESAREAP	Power coefficient in the lake area approximation function
PTOT_RAIN	Total monthly rainfall in the area of a WRUA
PTOT_AREA	Total area of a WRUA
PWATREQCR	Water requirements for achieving a maximum crop yield per period (*ETm*)
P_Y_MIN	Minimum yield level in %
P_Y_MAX	Maximum yield level in %
PY_R	Slope coefficient of the logistic yield approximation function
PWATRQFCT	Factor distributing seasonal water requirements to crop stages